PERGAMON

International Journal of Heat and Mass Transfer 45 (2002) 4367–4368

International Journal of HEAT and MASS TRANSFER

www.elsevier.com/locate/ijhmt

Technical Note

A note on the modeling of local thermal non-equilibrium in a structured porous medium

D.A. Nield *

Department of Engineering Science, University of Auckland, Private Bag 92019, Auckland, New Zealand

Received 24 January 2002; received in revised form 14 March 2002

Abstract

The traditional two-temperature thermal energy equations modeling local thermal equilibrium in a saturated porous medium are modified to model the case of convection when the medium has transverse structure. © 2002 Elsevier Science Ltd. All rights reserved.

This note is concerned with the modeling of heat transfer in a porous medium in which the solid material consists of parallel rods (or sheets), or alternatively in which the pore space consists of parallel tubes, in a situation in which local thermal equilibrium is not valid. This means that one has to allow for heat transfer between the solid and fluid phases. The traditional way of modeling this situation is to write down energy equations for the two phases (see, for example, Eqs. (2.11) and (2.12) of Nield and Bejan [1])

$$(1-\phi)(\rho c)_{\mathrm{s}}\frac{\partial T_{\mathrm{s}}}{\partial t} = (1-\phi)\nabla\cdot(k_{\mathrm{s}}\nabla T_{\mathrm{s}}) + h(T_{\mathrm{f}}-T_{\mathrm{s}}), \quad (1)$$

$$\phi(\rho c)_{\mathrm{f}}\frac{\partial T_{\mathrm{f}}}{\partial t} + (\rho c)_{\mathrm{f}}\mathbf{v}\cdot\nabla T_{\mathrm{f}} = \phi\nabla\cdot(k_{\mathrm{f}}\nabla T_{\mathrm{f}}) + h(T_{\mathrm{s}}-T_{\mathrm{f}}). \quad (2)$$

Here ϕ is the porosity and T_{f}, $(\rho c)_{\mathrm{f}}$, k_{f} and T_{s}, $(\rho c)_{\mathrm{s}}$, k_{s} are the temperatures, heat capacities and thermal conductivities in the fluid and solid phases, respectively, while $\mathbf{v}$ is the Darcy velocity and h is the fluid-to-solid heat exchange coefficient.

The reader should note that these expressions depend on the porosity but not on the precise way in which the fluid and solid phases are distributed within a representative elementary volume of the porous medium. Each term in these expressions represents a thermal energy per unit volume of the porous medium. Except for the transfer terms (the last in each equation) the flows within the separate phases are assumed to be autonomous. Clearly, if one adds the two equations then one gets an equation that governs the net heat transfer in the porous medium. In the case of local thermal equilibrium, $T_{\mathrm{f}} = T_{\mathrm{s}} = T$, and the combined equation is

$$(\rho c)_{\mathrm{eff}}\frac{\partial T}{\partial t} + (\rho c)_{\mathrm{f}}\mathbf{v}\cdot\nabla T = \nabla\cdot(k_{\mathrm{eff}}\nabla T), \quad (3)$$

where

$$(\rho c)_{\mathrm{eff}} = \phi(\rho c)_{\mathrm{f}} + (1-\phi)(\rho c)_{\mathrm{s}}, \quad (4)$$

$$k_{\mathrm{eff}} = \phi k_{\mathrm{f}} + (1-\phi)k_{\mathrm{s}}, \quad (5)$$

are the effective capacities and conductivities, respectively. It is noteworthy that k_{eff} is the weighted arithmetic mean of k_{f} and k_{s}. Since conductivity is the reciprocal of resistivity, Eq. (5) corresponds to the case where the resistivities of the fluid and solid phases are assumed to be in parallel. If they were assumed to be in series, then Eq. (5) would be replaced by

$$\frac{1}{k_{\mathrm{eff}}} = \frac{\phi}{k_{\mathrm{f}}} + \frac{(1-\phi)}{k_{\mathrm{s}}}. \quad (6)$$

In a general situation, the expression in Eq. (5) yields an upper bound on the value of k_{eff} while that in Eq. (6) yields a lower bound.

A rigorous two-equation model for heat transfer was presented by Quintard and Whitaker [2]. However, the

* Tel.: +64-9-3737-599; fax: +64-9-3737-468.
E-mail address: d.nield@auckland.ac.nz (D.A. Nield).

PII: S0017-9310(02)00138-2

results in that paper are presented in terms of integrals and these are not in a convenient form suitable for immediate application. Something simpler is desirable. Cheng and Hsu [3] have presented an analysis, for the case of local thermal equilibrium, which indicates that the simple model expressed by Eqs. (3)–(5) is valid if the effect of tortuosity (due to the undulating thermal path across the fluid–solid interface) is negligible.

The question raised here is how one can best adjust Eqs. (1) and (2) to the case of structured media for forced convection in, for example, a parallel-plate channel in which the solid material is layered, and is in the form of slabs parallel to the boundaries. In this case the fluid and the solid phases are in parallel for heat conduction in the longitudinal direction (x-direction) but in are in series for conduction in the transverse direction (y-direction). Thus the conduction term in Eq. (3) becomes

$$\nabla \cdot (k_{\text{eff}} \nabla T) = \frac{\partial}{\partial x}\left(k_1 \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_2 \frac{\partial T}{\partial y}\right), \tag{7}$$

where, from Eq. (6),

$$k_1 = \frac{k_f k_s}{\phi k_s + (1-\phi)k_f}, \tag{8}$$

and, from Eq. (5),

$$k_2 = \phi k_f + (1-\phi)k_s. \tag{9}$$

Working backwards, this suggests that for the case of local thermal non-equilibrium one should use

$$(1-\phi)(\rho c)_s \frac{\partial T_s}{\partial t} = (1-\phi)\left\{\frac{\partial}{\partial x}\left(k_s' \frac{\partial T_s}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_s \frac{\partial T_s}{\partial y}\right)\right\} + h(T_f - T_s), \tag{10}$$

$$\phi(\rho c)_f \frac{\partial T_f}{\partial t} + (\rho c)_f \mathbf{v} \cdot \nabla T_f = \phi\left\{\frac{\partial}{\partial x}\left(k_f' \frac{\partial T_f}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_f \frac{\partial T_f}{\partial y}\right)\right\} + h(T_s - T_f), \tag{11}$$

where

$$k_s' = k_f' = \frac{k_f k_s}{\phi k_s + (1-\phi)k_f}. \tag{12}$$

So far we have considered the extreme case. A less extreme situation is when the solid matrix of the porous medium is distributed in the form of rods with axes aligned in the longitudinal direction. In this case the fluid and solid phases are still in parallel for longitudinal conduction, but for transverse conduction they are neither in parallel nor in series. In this case it appears that no simple expressions for k_f' or k_s' are available, but one can still use the expressions given by Eq. (12) as bounds, and the values k_f and k_s as alternative bounds, on these two quantities. (Which one is a lower bound for a particular quantity, and which is an upper bound, depends on whether k_f is greater or less than k_s.)

Acknowledgements

The author is grateful to Dr A.V. Kuznetsov of North Carolina State University for drawing his attention to this subject.

References

[1] D.A. Nield, A. Bejan, Convection in Porous Media, second ed., Springer-Verlag, New York, 1999, p. 26.
[2] M. Quintard, S. Whitaker, Theoretical analysis of transport in porous media, in: K. Vafai (Ed.), Handbook of Porous Media, Marcel Dekker, New York, 2000, pp. 1–52.
[3] P. Cheng, P.C.-T. Hsu, Heat conduction, in: D.B. Ingham, I. Pop (Eds.), Transport Phenomena in Porous Media, Elsevier Science, Oxford, 1999, pp. 57–76.